Comment on "Applications for Deployed High Temperature Superconducting Coils in Spacecraft Engineering: A Review and Analysis" by J. C. Cocks et al.

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Abstract. *Cocks et al.* [1997] have incorrectly applied the theory of *Störmer* [1955] to a coil of superconducting wire that they propose can be used as an effective magnetic shield capable of deflecting harmful galactic cosmic rays (GCRs) and thereby protecting a region of space near the center of the coil. The results obtained by *Störmer* [1955] apply to an ideal dipole magnetic field. In order to apply this analysis to a circular coil of wire, the radius of the coil must be much smaller than the dimension of the region being shielded. Numerical simulations show that little shielding of the type obtained by *Störmer* [1955] is realized if this condition is violated. In addition, the shielded region demonstrated by *Störmer* [1955] is toroidal in shape rather than spherical as is implied by *Cocks et al.* [1997]. It can be shown that the shielding capacity of deployed coils decreases dramatically as the radius of the coil increases and the magnetic moment is held constant. Numerical simulations and the proper use of the analysis by *Störmer* [1955] demonstrate that magnetic shields consisting of deployed circular coils of wire are not practical for shielding energetic particles such as GCRs from spacecraft.

Keywords: magnetic spacecraft shielding — Störmer theory — cosmic rays — magnetic dipole approximation

1. Introduction

In their paper, *Cocks et al.* [1997] promote the idea of using a large-diameter coil of high-temperature superconducting wire that extends well beyond the confines of a spacecraft to produce a magnetic field that is intended to deflect hazardous energetic particles, such as galactic cosmic rays (GCRs), thereby protecting or shielding the spacecraft occupants from the harmful effects of the GCRs. The authors demonstrate several advantages of such a so-called deployed magnetic shield over other conventional passive and active shields. In particular, the significant reduction in energy required to generate a low intensity magnetic field with a large magnetic moment using a coil with a radius of up to 10 km, makes this strategy quite appealing and practical for long-duration missions, such as to Mars, where energy limitations are critical to success. An additional benefit attributed to deployed coils is the reduced magnetic field strength in close proximity to the spacecraft occupants. More traditional confined magnetic shields must be very intense ($\gg$1 T) in order to shield occupants from GCRs [e.g., *Parker*, 2005], however, the effects of such large magnetic fields on living cells are not entirely understood.

There is no disputing that the energy requirements and magnetic field intensity both decrease as the radius of a deployed coil increases and the magnetic moment M remains constant. It is, however, questionable as to whether such a magnetic field configuration provides the shielding that is claimed by *Cocks et al.* [1997]. We show that *Cocks et al.* [1997] have made several mistakes in their analysis and that large-diameter deployed coils provide little, if any, protection from GCRs. Numerical simulations in a companion paper support these claims [*Shepherd and Kress*, 2006].

2. Discussion

Cocks et al. [1997] rely on analysis by *St¨ormer* [1955] in order to describe a region that is shielded from particles below a certain energy. The size of the shielded region is characterized by the length scale C_{st}, which is sometimes known as the Störmer length, and is given by

$$C_{st} = \sqrt{\frac{Mq\mu_0}{4\pi\gamma mv}} \tag{1}$$

where M is the magnetic dipole moment, q is the charge of the particle, $\gamma = (1 - v^2/c^2)^{1/2}$ is the relativistic correction factor, m is the rest mass of the particle, v is the speed of the particle, c is the speed of light, and SI units are used throughout. Note that *Cocks et al.* [1997] write equation (1) in terms of the kinetic energy (KE) of a particle, but the expression is only valid for particles with $v \ll c$, or $\gamma \simeq 1$. For reference, a 10 GeV, singly-ionized iron atom (Fe^+) (a moderately energetic GCR) moves at greater than half the speed of light and for which $\gamma \simeq 1.2$.

The analysis by *Cocks et al.* [1997] seems to imply that there exists a spherical region given by C_{st} from which particles below a certain energy are shielded. The forbidden or shielded region that is derived by *St¨ormer* [1955] is in fact toroidal in shape and is somewhat smaller in dimension than C_{st}. The boundary of this region is defined by the equation

$$r = C_{st}\ \frac{\cos^2\lambda}{1+\sqrt{1+\cos^3\lambda}}\ , \tag{2}$$

where λ is the magnetic latitude ($\pi/2-\theta$) and r is the radial distance from the center of the magnetic dipole in spherical coordinates. It is important to note that the magnetic field of an ideal dipole is used in the derivation of equation (2). At the equator ($\lambda = 0$) $r \simeq 0.4C_{st}$, however at the poles ($\lambda = \pm\pi/2$), $r = 0$. Particles of the given energy therefore have access to the region near the origin through the polar cusp-like regions of the magnetic field. At the very least, the shape of the shielded region should be a factor to consider in designing the shape of the spacecraft and its position with respect to the field.

A more serious mistake in the analysis by *Cocks et al.* [1997] is that the authors assume that the magnetic field associated with a coil of radius a and n wires carrying current I (a magnetic dipole moment $M = nI\pi a^2$) is equivalent to the magnetic field of a pure

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dipole with the same M. Such an approximation, and thus the application of the result by *St¨ormer* [1955] (hereafter we refer to the analysis and shielding of this work as Störmer theory and Störmer-like shielding), is only valid for distances (measured from the center of the current loop) that are much greater than the radius of the loop [e.g., *Reitz et al.*, 1980]. We can therefore only expect the Störmer theory to strictly apply in situations when all points along a particle trajectory are at distances much greater that the radius of the loop and therefore experience the dipole magnetic field. A somewhat relaxed but necessary requirement is that $C_{st} \gg a$, i.e., the dimensions of the shielded region are much larger that the coil radius. In this case particles entering along the polar axis may still gain access to the near (non-dipolar) field, however, we have found numerically that the requirement that $C_{st} \gg a$ provides a reasonably good approximation for Störmer-like shielding [*Shepherd and Kress*, 2006].

Figure 1 illustrates the difference between a pure dipole field with magnetic dipole moment $M = M\hat{z}$ ($M = 1.1 \times 10^{13}$ A m^2) and the magnetic field of a 1 km loop of current (I) with the same magnetic moment (M). For this example the current was chosen to be $I = 20$ kA, the number of turns of wire to be $n = 175$, and the radius of the coil $a = 1$ km. The magnitude of the magnetic field ($|B|$) is shown in units of Tesla on a logarithmic scale, indicated by the greyscale contours, and several representative field lines are also shown. The field in both cases is axisymmetric about the z-axis.

Two things are immediately apparent in Figure 1, (1) the magnitude of the dipole field is unbounded at the origin, while the field due to the coil is finite there. (2) The fields in the $z = 0$ plane point in the opposite direction for distances that are inside the radius of the coil. Clearly a particle whose path comes within the limits of Figure 1 experiences a force, and therefore a different trajectory, that depends on the particular magnetic field geometry. The differences are noticeably more pronounced for trajectories approaching the origin along the polar axis.

Numerical simulations have been performed on particles of various masses and energies in magnetic fields corresponding to a pure dipole magnetic field and the field due to a current loop [*Shepherd and Kress*, 2006]. Figure 2 shows the results of computing the trajectories of 50,000 singly-ionized iron atoms (Fe^{+}) with kinetic energy 1 GeV and initial velocities directed toward the origin using a standard numerical technique. Each point in Figure 2 represents the location of nearest approach to the origin of each of the particle trajectories. The toroidal shielded region given by equation (2) is shown as a solid line while the Störmer length (C_{st}) given by equation (1) is shown as a dashed line further from the origin.

Note the excellent agreement between the shielded region predicted by Störmer theory for a dipole field in Figure 2a and the lack of shielding and protection from the given particles that is provided by the 1 km radius current loop with the same magnetic moment M, shown in Figure 2b. For this example, the outer limit of the shielded region given by equation (2) is ∼74 m. It is clear that trajectories at all latitudes bring particles inside the 1 km radius of the coil. Two things happen to these particles in the presence of the field due to the coil, (1) they experience forces from a field configuration that is much different from a dipole field, and (2) the strength of the field (and the corresponding force) is significantly weaker than it would be for the dipole. The result is that for particles with an energy that causes them to violate the condition that $C_{st} \gg a$, Störmer-like shielding does not occur.

Assuming that the condition for the size of the coil is met for Störmer theory to be valid (i.e., $C_{st} \gg a$) it is possible to show that for fast moving particles with $\gamma \gg 1$, the energy of the particles (KE) that are shielded from the toroidal region given by equation (1) can be written as

$$KE \simeq \gamma mc^2 \sim \frac{1}{a^2} \tag{3}$$

That is, the energy of the particles that are shielded from the desired region are inversely proportional to the square of the radius of the coil. The reduction is even greater for slower moving (lower energy) particles with $\gamma \approx 1$, such as those for which the analysis of *Cocks et al.* [1997] applies. In this case it is easy to show that

$$KE \simeq \frac{1}{2}mv^2 \sim \frac{1}{a^4} \tag{4}$$

So while the energy and mass requirements of a deployed coil decrease with increasing radius as shown by *Cocks et al.* [1997], one must also consider that the energy of the particles that are shielded from the desired region also decreases. The implication to the feasibility of magnetic shields using deployed coils is quite serious.

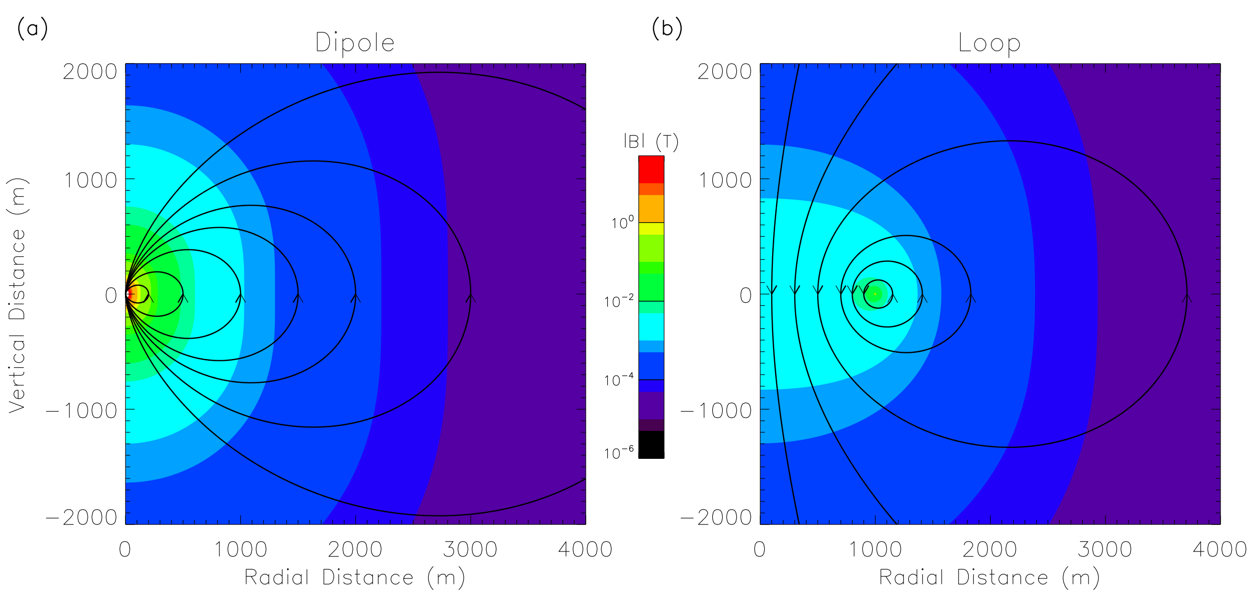

Figure 1. Magnetic field strength shown in greyscale and several representative magnetic field lines for a (a) pure dipole field and a (b) loop of current with a radius of 1 km. The magnetic moment M is that same for both configurations. Note that both the dipole and loop fields exceed the maximum scale value; near the origin for the dipole and near the wire for the loop.

Shielding high-energy GCRs using such a system requires that the diameter of the coil be smaller than the region being shielded, which in turn implies that large currents and large magnetic fields are located in close proximity to the spacecraft inhabitants – a serious problem that deployed shields are intended to remedy.

To illustrate this point, we assume that the following condition holds $a \leq \epsilon C_{st}$ where ϵ is a small number ($\ll 1$) and that Störmer theory holds, i.e., Störmer-like shielding occurs. It is then possible to write the following expression for the minimum current required to shield particles from the desired region:

$$nI \geq \frac{R}{30\pi\epsilon^2} \tag{5}$$

where $R \equiv \gamma mvc/q$ is known as the rigidity of a particle with rest mass m, charge q, and speed v. A handy rule-of-thumb is obtained by noting that equation (5) simplifies to $nI \geq R$ for $\epsilon \simeq 1/9.7$; a value which has been shown to be satisfactory in numerical simulations [*Shepherd and Kress*, 2006]. Note also that equation (5) is independent of the size of the coil and thus the shielded region. As pointed out by *Cocks et al.* [1997], however, the energy stored in a coil of wire increases by a factor that is greater than a.

In order to investigate the feasibility of Störmer-like shielding using a deployed coil as suggested by *Cocks et al.* [1997], we apply equation (5) to various energetic GCRs. Table (1) shows the combined current (nI) that is required to shield Fe^{+} and H^{+} particles of energies between 1 MeV and 100 GeV assuming that equation (5) holds for $\epsilon = 1/9.7$. As an example, assuming 1000 turns of wire in a coil, the current required to shield a moderately energetic 50 GeV Fe^{+} atom is ∼89 MA. The magnetic field strength at the center of 100 m radius coil carrying this amount of current is >550 T; almost certainly lethal to any nearby occupants.

Table 1. Minimum total current (nI) necessary in a coil to achieve Störmer-like shielding from particles (Fe^{+} and H^{+}) having the given energy (KE) and corresponding rigidity (R) assuming that equation (5) holds with $\epsilon = 1/9.7$.

KE	Fe^{+} R [GV] and nI [GA]	H^{+} R [GV] and nI [GA]
1 MeV	0.325	0.0435
10 MeV	1.03	0.138
100 MeV	3.26	0.446
500 MeV	7.30	1.10
1 GeV	10.3	1.70
10 GeV	34.1	11.0
50 GeV	88.5	51.2
100 GeV	144.	102.

If we investigate the shielding potential using a deployed coil powered with transistor radio batteries (∼2 A), as suggested by *Cocks et al.* [1997], we see that the energy of the particles shielded with such a configuration is exceedingly low. In fact, a proton moving at ∼1.5 km s^{-1} has the maximum energy (∼1.2 × 10^{-2} eV) that can be expected to be shielded from the toroidal Störmer-like region using the suggested configuration. For reference, a solar wind proton moving at a typical speed of 440 km s^{-1} has an energy of 1 keV. It is also worth pointing out that the magnetic field strength at the center of a 100 m coil with a single loop of wire carrying 2 A is ∼12 nT, or comparable in magnitude to the interplanetary magnetic field at a 1 AU during nominal solar wind conditions.

3. Summary

Of recent interest to interplanetary space travel is the ability to shield the inhabitants of a spacecraft from the harmful effects of galactic cosmic ray (GCRs). Many novel techniques have been suggested to accomplish this task [e.g., *Townsend*, 2000]. One particular example consists of a superconducting coil of wire deployed in such a manner that it extends well beyond the confines of the spacecraft and provides a magnetic field capable of deflecting harmful particles from the inhabited region. *Cocks et al.* [1997] promote this technique as a viable technological solution to the problem of shielding interplanetary travelers from GCRs. The authors demonstrate that such a deployed magnetic shield is superior to other techniques in several aspects; most notably the reduced energy required to maintain a magnetic moment M with ever increasingly large coils and the reduced magnetic field strength in the proximity of the shielded region.

In their analysis, *Cocks et al.* [1997] rely on theory developed by *Störmer* [1955] for a dipole magnetic field configuration. The authors mistakenly assume (1) that the shielded region is spherical rather than toroidal, and (2) that the field of a dipole is equivalent to that of a current loop. In order to apply Störmer theory to a deployed magnetic shield, the condition that the radius of the coil is much smaller than the outer dimension of the toroidal region being shielded must be satisfied; that is, $a \ll C_{st}$, where a is the radius of the coil and C_{st} is given by equation (1).

Furthermore, it can be shown that the energy of the particles that can be shielded from a given region using a coil of radius a with a constant magnetic moment M falls off as a^{-2} or a^{-4} for relativistic and non-relativistic particles, respectively. Likewise, if Störmer-like shielding is to apply, the magnitude of the total current nI is then proportional to R, the rigidity of the particle. That is, in order to shield increasingly energetic particles, a stronger current requiring more energy is required.

The reality of the situation is that large magnetic fields are necessary to provide sufficient force to deflect energetic particles, such as GCRs, from a given region. Deployed magnetic shields have reduced energy requirements and magnetic field strengths relative to confined systems, therefore, they also have a significantly reduced capacity for protecting the occupants of the desired shielded region. Our analysis leads us to believe that magnetic shields consisting of deployed coils are not a practical solution to shielding occupants of a spacecraft from energetic particles such as GCRs.

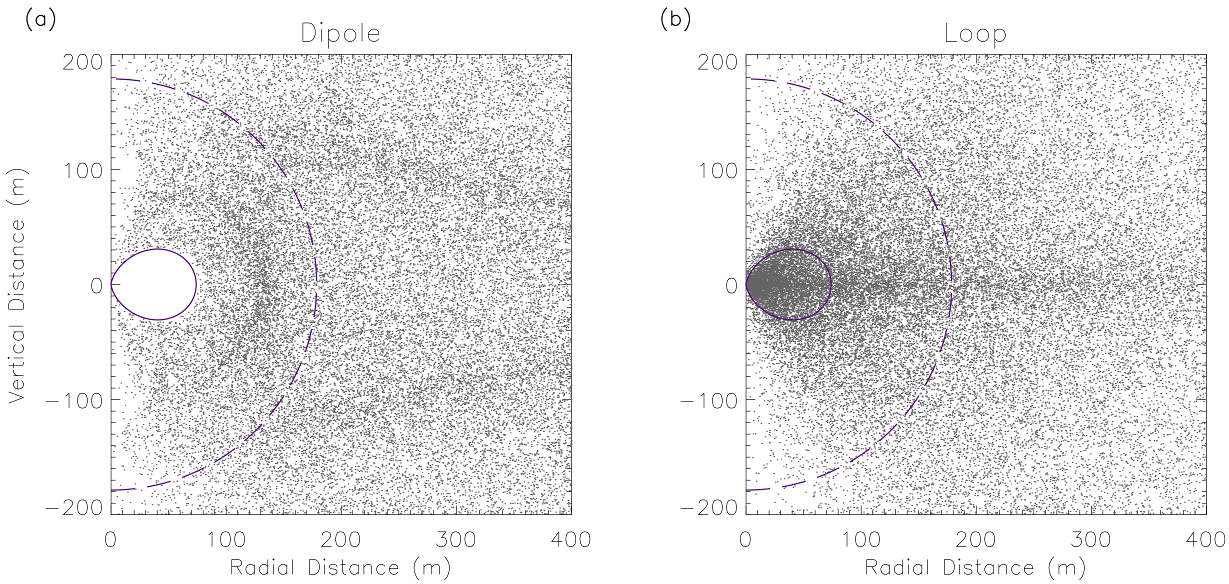

Figure 2. Closest approach to the origin of 50,000 Fe^+ particles with energy 1 GeV in the magnetic field of (a) a pure dipole and (b) a 1 km radius current loop, both with dipole moment $M = 1.1 \times 10^{13}$ A m^2. The shielded region given by equation (2) is shown by the solid inner line and the region defined by C_{st} in equation (1) is shown by the outer dashed line.

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