Testing the Hill model of transpolar potential with Super Dual Auroral Radar Network observations

S. G. Shepherd, J. M. Ruohoniemi, and R. A. Greenwald

Received 3 May 2002; revised 28 June 2002; accepted 31 July 2002; published 2 January 2003.

We use a data set consisting of periods for which the transpolar ionospheric potential ($\Phi_{pc}$) is well-determined by Super Dual Auroral Radar Network (SuperDARN) data to test the Hill model. The Hill model, as formulated by Siscoe et al. [2002], specifies $\Phi_{pc}$ as a function of solar wind speed and ram pressure, the interplanetary magnetic field, the reconnection electric field ($E_r$), and the ionospheric conductance ($\Sigma$). The periods used in our study were identified as times when the interplanetary electric field was quasi-stable for and SuperDARN coverage was sufficient to determine $\Phi_{pc}$. SuperDARN-determined $\Phi_{pc}$ is compared to $\Phi_{pc}$ determined using the Hill model ($\Phi_{pc,Hill}$) for 1317 10-min periods. A minimum in the root-mean-square difference between ($\Phi_{pc,Hill}$) ($E_r$) and ($\Phi_{pc,Hill}$) ($E_r$) is achieved when $\Sigma = 23$ S and a constant potential, $\Phi_0 = 17$ kV, are used. Some aspects of the data agree very well for these values of $\Sigma$ and $\Phi_0$, including the mean value of $\Phi_{pc}(E_r)$ and that both datasets clearly indicate saturation at higher values of $E_r$. The ram pressure dependence of ($\Phi_{pc,Hill}$), however, is inconsistent with that of ($\Phi_{pc,SD}$) and suggests that $\Sigma$ should be lower than 23 S. There is also significantly more variability in ($\Phi_{pc,SD}$) for all values of $E_r$ than the Hill model predicts.

INDEX TERMS: 2463 Ionosphere: Plasma convection; 2431 Ionosphere: Ionosphere/magnetosphere interactions (2736); 2784 Magnetospheric Physics: Solar wind/magnetosphere interactions; 2437 Ionosphere: Ionospheric dynamics; 2411 Ionosphere: Electric fields (2712).


1. Introduction

[1] The transpolar potential or cross polar cap potential ($\Phi_{pc}$) is the total variation in the high-latitude ionospheric electric potential and is an important indicator of the amount of energy flowing into and through the magnetosphere-ionosphere (M-I) system. It is a convenient parameter often used to compare different methods of calculating electric fields in the high-latitude ionosphere.

[2] Many techniques have been developed to determine a relationship between the upstream solar wind and interplanetary magnetic field (IMF) conditions and $\Phi_{pc}$. Some of these include spacecraft measurements of the convecting plasma [Heppner, 1972; Reiff et al., 1981; Doyle and Burke, 1983; Rich and Hairston, 1994; Boyle et al., 1997; Weimer, 2001], assimilation of ground and satellite measurements such as the Assimilative Mapping of Ionospheric Electrodynamics (AMIE) technique [Richmond and Kamide, 1988], fitting ionospheric line-of-sight (LOS) convection velocities from ground-based radars to functional forms of the electrostatic potential [Ruohoniemi and Baker, 1998], and global magnetospheric modeling codes [Fedder and Lyon, 1987; Raeder et al., 2001; Ridley, 2001].

[3] Siscoe et al. [2002] have proposed a formulation of a theoretical model, the Hill model [Hill et al., 1976], which takes into account M-I coupling by Region 1 currents that act to effectively reduce the strength of the magnetic field at the magnetopause merging region as the solar wind electric field ($E_{sw}$) increases. The resulting feedback limits the amount of reconnection at the dayside magnetopause thereby limiting the magnitude of $\Phi_{pc}$, a behavior known as saturation [e.g., Russell et al., 2001]. Siscoe et al. [2002] compared the Hill model with results from a global magnetospheric MHD model, the Integrated Space Weather Model (ISM), and found good agreement in terms of both Region 1 currents and $\Phi_{pc}$. In this letter we report the first test of the transpolar potential from the Hill model ($\Phi_{pc,Hill}$) against direct measurements of the potential from Super Dual Auroral Radar Network (SuperDARN) observations ($\Phi_{pc,SD}$).

2. Procedure

[5] Shepherd et al. [2002] identified quasi-stable periods of the Kan–Lee electric field, $E_{KL} \equiv VB_T \sin^2(\theta/2)$, where $V$ is the solar wind speed, $B_T$ is the transverse IMF, $BT = (B_x^2 + B_y^2)^{1/2}$, and $\theta$ is the IMF clock angle; $\cos^{-1}(B_y/B_T)$ [Kan and Lee, 1979]. We break from tradition at this point and change notation from $E_{KL}$ to $E_r$. The reason for our switch is that Sonnerup [1974] first correctly demonstrates that the upper limit of the reconnection electric field is proportional to $\sin^2(\theta/2)$ and attributed this dependence to Petschek [1964, 1966] and Nishida and Maezawa [1971]. We therefore chose to refer to the quantity $V B_T \sin^2(\theta/2)$ as the reconnection electric field, $E_r$.

[6] The periods identified by Shepherd et al. [2002] were observed by the Advanced Composition Explorer (ACE) between February 1998 and December 2000, and were required to be stable with respect to $E_r$ to within a specified degree for $\geq 40$ min. The reason for the stability criteria was to minimize the impact of the inherent uncertainties in predicting the lag time of observation of the solar wind and IMF at ACE to impact in the high-latitude ionosphere [e.g., Ridley et al., 1998]. A further attempt was made to
minimize this impact by throwing out the first and last 10-min periods of each ≥40-min event.

In addition to the stability requirement, these periods also satisfy the criterion that \( \Phi_{pc} \) determined using the Johns Hopkins University Applied Physics Laboratory Fitting technique or APL Fit [Ruhonioni and Baker, 1998; Shepherd and Ruohoniemi, 2000], were suitably well-defined by the SuperDARN LOS Doppler velocity data. Further details of the selection criteria are provided by Shepherd et al. [2002].

We use a subset of 1317 10-min periods from the Shepherd et al. [2002] data set for this study. Because the statistics are greatest in the range \( E_r ≤ 30 \text{ kV R}_{E}^{-1}(≈4.7 \text{ mV m}^{-1}) \), we use only the periods which fall in this range. Appropriately time lagged 10-min averages of the ACE/SWEPAM solar wind speed (\( V \)) and mass density (\( \rho \)), and the ACE/MAG IMF (\( \vec{B} \)) are used to derive \( \Phi_{pc} \). \( \Phi_{pc} \) is also computed for the Boyle model [Boyle et al., 1997] for comparison. \( \Phi_{pc} \) for these models are then directly compared to the SuperDARN-derived transpolar potential \( \Phi_{pc}^{SD} \).

The formulation of the Hill model we use in this study is given by Siscoe et al. [2002]

\[
\Phi_{pc}^{Hill} = \frac{57.6 V_{sw} \rho F(\theta)}{p^2 + 0.0125 \Sigma E_{sw} F(\theta)}
\]

where \( E_{sw} = VB_T \) is the solar wind electric field with \( V \) being the solar wind speed, \( B_T \) the transverse IMF, and \( p \) is the solar wind ram pressure. \( F(\theta) \) is the IMF clock angle dependence of reconnection at the magnetopause, taken to be \( \sin^2(\theta/2) \) following Siscoe et al. [2002]. A relation for \( \xi \), a factor that depends on the geometry of the currents flowing into the ionosphere, is given as 4.45 − 1.08 log \( \Sigma \) by Siscoe et al. [2002], where \( \Sigma \) is the ionospheric conductance and is assumed to be uniform. We note that the choice of \( F(\theta) \) allows us to rewrite equation (1) as

\[
\Phi_{pc}^{Hill}(\Sigma) = \frac{57.6 E_{sw} \rho F(\theta)}{p^2 + 0.0125 \xi \Sigma E_{sw}}
\]

where \( E_r \) is the reconnection electric field discussed earlier in section 2.

3. Data and Discussion

[10] Figure 1a shows \( \Phi_{pc}^{SD} \) plotted against \( E_r \), as circles and \( \Phi_{pc}^{Hill}(\Sigma = 23 \text{ S}) + 17 \text{ kV} \) as triangles (a best-fit solution to \( \Phi_{pc}^{SD} \)). For each model a linear fit to \( \Phi_{pc} \) in a 5-kV \( R_{E}^{-1} \) window of \( E_r \) is shown for each unit value of \( E_r \) in kV \( R_{E}^{-1} \). We also plot just the linear fits of several other models for comparison: \( \Phi_{pc}^{Hill}(\Sigma = 2 \text{ S}) \), \( \Phi_{pc}^{Hill}(\Sigma = 44 \text{ S}) \), and \( \Phi_{pc} \) for the Boyle model. Figure 1b shows the distribution of \( E_r \) for the data set used in this study with a peak in the distribution near 13 kV \( R_{E}^{-1} \)(≈2.0 \text{ mV m}^{-1}). We show the mean and standard deviation of the windowed, linear fits to \( \Phi_{pc}(E_r) \) in Figure 2a. Because there is a great deal of variability in \( \Phi_{pc} \) (particularly in \( \Phi_{pc}^{SD} \)) for nearly all ranges of \( E_r \) (Figure 2b).

[11] The two Hill models, \( \Phi_{pc}^{Hill}(\Sigma = 2 \text{ S}) \) and \( \Phi_{pc}^{Hill}(\Sigma = 44 \text{ S}) \), represent the minimum and maximum values of \( \Sigma \) used by Siscoe et al. [2002] and serve to illustrate the extremes of this model. Another Hill model, \( \Phi_{pc}^{Hill}(\Sigma = 23 \text{ S}) + 17 \text{ kV} \), shown in Figure 2a and Figure 3 corresponds to the best-fit solution of \( \Phi_{pc}(\Sigma) + \Phi_0 \) to \( \Phi_{pc}^{SD} \). The linear fits to these data match nearly perfectly with those of over the entire range \( 0 ≤ E_r ≤ 30 \text{ kV} R_{E}^{-1} \).

[12] The reason for including the constant potential term \( \Phi_0 \) in the best-fit solution is that one expects a non-zero minimum potential even for very low \( E_{sw} \). Viscous magnetosheath convection, reconnection in the magnetotail, or ionospheric processes generally prevent \( \Phi_{pc} \) from ever reaching zero. Because \( \Phi_{pc}^{Hill} = 0 \) for \( E_{sw} = 0 \), we add the constant \( \Phi_0 \) to \( \Phi_{pc}^{Hill}(\Sigma) \) when comparing to \( \Phi_{pc}^{SD} \), and attribute it to the effects of these processes.

[13] In order to determine the best-fit \( \Phi_{pc}^{Hill}(\Sigma) + \Phi_0 \) to \( \Phi_{pc}^{SD} \), total root-mean-square (RMS) differences between the two data sets were calculated in two ways. Figure 3a shows unit contours of the total RMS difference for the 1317

---

Figure 1. (a) \( \Phi_{pc} \) determined using APL Fit (\( \Phi_{pc}^{SD} \)) and the best-fit Hill model (\( \Phi_{pc}^{Hill} \), shown in Figure 4b) plotted against \( E_r \). A linear fit to a 5-kV \( R_{E}^{-1} \) window is shown for these data and for several other comparison models: the Boyle model, \( \Phi_{pc}^{Hill}(\Sigma = 2 \text{ S}) \), and \( \Phi_{pc}^{Hill}(\Sigma = 44 \text{ S}) \). (b) Distribution of events in \( E_r \).

Figure 2. \( \Phi_{pc}^{Hill} \) Mean and standard deviation of \( E_r \)-windowed, linear fits to the data shown in Figure 1a.
Figure 3. Unit contours of RMS differences between $\Phi_{pc}^{Hill}(E_r)$ and $\Phi_{pc}^{SD}(E_r)$, in $\Sigma$, $\Phi_0$ space, for (a) all data points (Figures 2a and 2b) $E_r$-windowed, linear fits to the data (Figure 3). The minimum total RMS values are located at the intersection of the dotted lines.

periods shown in Figure 1a. This solution corresponds to a fit to all the data points. A minimum occurs when $\Sigma = 27$ S and $\Phi_0 = 22$ kV, but it is clear that there is a family of solutions for which the RMS difference is within a few kV of this minimum. This fact is partly due to the distribution of data points shown in Figure 1b. Figure 3b shows unit contours of the total RMS difference between the $E_r$-windowed, linear fits shown in Figures 1a and 2a, and has a well-defined minimum. We therefore consider this, $\Phi_{pc}^{Hill}(\Sigma = 23$ S $)+ 17$ kV, the best-fit solution to $\Phi_{pc}^{SD}$ for these data, and note there is a possible range of solutions as shown in Figure 4a.

The value of $\Phi_0 = 17$ kV seems reasonable for the minimum transpolar potential. For this data set the lowest value of $\Phi_{pc}$ for the Boyle model is $\sim 20$ kV and the lowest value of $\Phi_{pc}^{SD}$ is $\sim 18$ kV. Some studies report $\Phi_0 = 22$–39 kV [e.g., Reiff et al., 1981] but these values were obtained by a linear fit to the data and lower values were clearly present.

While $\Phi_0$ is in good agreement with other studies, the value of ionospheric conductance, $\Sigma = 23$ S, is quite high. A typical value of uniform ionospheric conductance used in global magnetospheric MHD models is 5 S, and a few S is not unusual [e.g., Ridley, 2001]. Even if one considers the range of $\Sigma$ for the family of solutions in Figure 4, the minimum $\Sigma$ for these solutions is greater than 10 S, which is still quite high.

In order to be sure that the values of $\Phi_{pc}^{SD}$ at higher values of $E_r$ (>15 kV Re$^{-1}$) were not overly biasing the best-fit solution, RMS differences were calculated (but not shown) over the range 5 $\geq E_r$ $\geq 15$ kV Re$^{-1}$ where the statistics are greatest and values of $\Phi_{pc}^{SD}$ are most certainly well-defined [Shepherd et al., 2002]. The best-fit solution for this limited range is $\Phi_{pc}^{Hill}(\Sigma = 20$ S $)+ 16$ kV, which is quite similar to the fit for the whole range ($0 \geq E_r \geq 30$ kV Re$^{-1}$), suggesting that $\Phi_{pc}^{SD}$ at large $E_r$ does not bias the determination of $\Sigma$.

The ram pressure dependence of $\Phi_{pc}^{Hill}$ can also be compared to that of $\Phi_{pc}^{SD}$. Figure 4 shows $\Phi_{pc}^{SD}(\rho)$, the best-fit $\Phi_{pc}^{Hill}(\rho)$, and the two extreme-value Hill models, $\Phi_{pc}^{Hill}(\Sigma = 2$ S) and $\Phi_{pc}^{Hill}(\Sigma = 44$ S). The data have been binned in 5-kV Re$^{-1}$ intervals of $E_r$ and plotted in Figures 4a–f along with 1-$\sigma$ error bars. The horizontal dotted lines represent the average $\Phi_{pc}^{SD}$ for each range of $E_r$.

The general trend of increasing $\Phi_{pc}^{SD}$ with increasing $E_r$ is obvious from the panels in Figure 4. Also obvious is the lack of any $\Phi_{pc}^{SD}$ dependence on $\rho$ for any range of $E_r$. Unlike $\Phi_{pc}^{Hill}$ for the best-fit solution shows a very clear $\rho$-dependence and very narrow distributions, i.e., small $\sigma$. The $\rho$-dependence becomes more pronounced at higher values of $E_r$. On the other hand, $\Phi_{pc}^{Hill}(\Sigma = 2$ S) shows no $\rho$ dependence and the $\sigma$s are comparable to those of $\Phi_{pc}^{SD}$, suggesting that a lower value of $\Sigma$ better matches $\Phi_{pc}^{SD}(\rho)$.

Of course, the $E_r$ dependence of $\Phi_{pc}^{Hill}(\Sigma = 2$ S) is inconsistent with $\Phi_{pc}^{SD}$ – i.e., the value of $\Phi_{pc}^{Hill}$ is too high.

One possible explanation for the relatively high value of $\Sigma$ that is indicated by this comparison is that $\Phi_{pc}^{Hill}$ on average is somewhat lower than inferred by some other techniques. For example, drift meter measurements from a low altitude satellite such the Dynamics Explorer-2 report that $\Phi_{pc} = 110$ kV, and possibly higher, for IMF $E_r = -10$ nT with solar wind speed $V = 450$ km s$^{-1}$ ($E_r = -30$ kV Re$^{-1}$) [Weimer, 2001]. The mean $\Phi_{pc}^{SD}$ for this value of $E_r$ is $\sim 75$ kV. It should be noted that $\Phi_{pc}^{SD}$ does exceed 100 kV, but there is also a great deal of variability over the entire range of $E_r$, perhaps causing the mean to be lower than expected.

Although care was taken in selecting periods for this study, there are instances when the APL Fit technique underestimates $\Phi_{pc}$. During extended periods of large $E_r$ the favorable coupling between the solar wind and magnetosphere causes the lower latitude of the convection region to expand equatorward. If the convection region becomes too expanded the propagation conditions required to achieve perpendicularly and detect backscatter prevent the radars from observing the entire convection region. In this situation $\Phi_{pc}^{SD}$ underestimates the true $\Phi_{pc}$. It is possible that some of these periods exist in our study, however, their occurrence is expected at greater frequency for larger values of $E_r$. Because the RMS differences over the range 5 $\leq E_r$ $\leq 15$ kV Re$^{-1}$ also suggest a higher value of $\Sigma$, it is unlikely that a systematic underestimation of $\Phi_{pc}$ due to this scenario is the sole source of the discrepancy.
Some of the variability in \( \Phi_{SD}^{PC}(E_r) \) that is observed may be due to variations in the data coverage or variability in the solar wind, despite efforts to minimize uncertainties in determining \( \Phi_{SD}^{PC} \) and the associated geoeffective solar wind and IMF. However, it is also possible that variability of this magnitude can only be explained by internal effects and that a model of the ionospheric potential requires detailed knowledge of the coupling between the magnetosphere and ionosphere and its time history. Another possible explanation for the relatively high value of \( \Sigma \) suggested by our comparison is, therefore, that the Hill model, as formulated by Siscoe et al. [2002], needs some modification to better match the SuperDARN results.

One possibility is to adjust the parameter \( \xi \) in equation (2). Siscoe et al. [2002] determined the empirical expression for \( \xi \) given in section 2, from a fit to the results obtained from ISM and acknowledge that there is some flexibility in the value of this parameter. If we chose \( \Sigma = 5 \) S and \( \Phi_0 = 17 \) kV, then \( \Phi_{PC}^{Hill}(\Sigma, \xi = 10) + \Phi_0 \) is within 1-\( \sigma \) of \( \Phi_{SD}^{PC}(E_r) \) for \( E_r \leq 20 \) kV \( R_E^{-1} \), and within 2-\( \sigma \) for the entire range, \( 0 \leq E_r \leq 30 \) kV \( R_E^{-1} \). It is unclear whether \( \xi \) is 10 is a reasonable value since it is much larger than both the value of \( \sim 3.7 \) obtained by the experimental fit and the maximum value of 4.2 used in their study. For comparison, \( \Phi_{SD}^{Hill}(\Sigma = 5 \) S, \( \xi = 3.7) + 17 \) kV is with 1-\( \sigma \) of \( \Phi_{SD}^{PC}(E_r) \) only for \( E_r \leq 4 \) kV \( R_E^{-1} \), and within 2-\( \sigma \) for \( E_r \leq 9 \) kV.

It is also possible that other assumptions used to derive equation (1) are incorrect. For instance, the assumption of a uniform global conductivity may be inadequate, or the manner in which ionospheric conductivity mediates M-I coupling may need to be re-examined.

4. Conclusions

We have reported on the first experimental test of the Hill model, using determinations of \( \Phi_{PC} \) from SuperDARN observations. A large data set of quasi-stable IMF periods and suitable SuperDARN coverage was used to compare \( \Phi_{PC} \) of the Hill model (\( \Phi_{PC}^{Hill} \)) with that derived from the SuperDARN measurements (\( \Phi_{PC}^{SD} \)). The Hill model incorporates feedback from the Region 1 current system and correctly predicts saturation of the transpolar potential at high values of the solar wind electric field. Comparison with \( \Phi_{PC}^{SD} \), however, reveals some discrepancies.

The best-fit solution of the Hill model to \( \Phi_{SD}^{PC}(E_r) \) occurs when the constant ionospheric conductance (\( \Sigma \)) is set to 23 S and an added minimum potential (\( \Phi_0 \)) is set to 17 kV. The value of \( \Phi_0 \) is in reasonable agreement with the SuperDARN observations and that reported in other studies, however, the value of \( \Sigma \) is higher than expected. The ram pressure dependence of the Hill model also suggests that a lower value of \( \Sigma \) may be more appropriate. While the mean of \( \Phi_{SD}^{PC}(E_r) \) is somewhat lower than reported by others, it is unlikely that a systematic underestimation of \( \Phi_{PC} \) occurs over the entire range of \( 0 \leq E_r \leq 30 \) kV \( R_E^{-1} \), and is soley responsible for the large \( \Sigma \). Most likely some of the assumptions Siscoe et al. [2002] used to determine equation (1) need modification.

The Hill model is certainly an advance in the sophistication of representing \( \Phi_{PC} \) in terms of measurable quantities. Saturation of the transpolar potential is a salient feature of this model missing in many others. Further study, however, is necessary to determine the cause of the observed differences with the SuperDARN results.

Acknowledgments. This work was supported by NSF grant ATM-9812078 and NASA grant NAGS-8361. Operation of the Northern Hemisphere SuperDARN radars is supported by the national funding agencies of the U.S., Canada, the U.K., and France. The Hill model formulation was kindly provided by Dr. George Siscoe. We gratefully acknowledge the ACE/MAG instrument team, the ACE/SWEPAM instrument team, and the ACE Science Center for providing the ACE level 2 data.

References


S. G. Shepherd, Thayer School of Engineering, Dartmouth College, 8000 Cummings Hall, Hanover, New Hampshire 03755-0000, USA. (s.shepherd@dartmouth.edu)
R. A. Greenwald and J. M. Ruhoniemi, The Johns Hopkins University Applied Physics Laboratory, 11100 Johns Hopkins Road, Laurel, MD 20723. (raymond.greenwald@jhuapl.edu; mike.ruohoniemi@jhuapl.edu)